The Contingent Claims Approach to Corporate Vulnerability Analysis: Estimating Default Risk and Economy-wide Risk Transfer


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Abstract

In this paper we examine the ability of the contingent claims approach (CCA) to identify corporate sector and economy-wide vulnerabilities. We apply the Moody’s M/Risk model, which uses aggregated CCA principles, to assess vulnerabilities retroactively in two historical country cases. The results indicate that the method holds promise in identifying corporate sector vulnerabilities and estimating the associated value of risk transfer across interrelated balance sheets of the corporate, financial, and public sector.

JEL Classification Numbers: G13, G32, G34.

Keywords: Contingent claims approach, Corporate sector vulnerability, Credit risk.

I. INTRODUCTION

Many studies have documented the importance of monitoring weaknesses in the corporate sector and their impact on the wider financial system. The motivation for close monitoring derives from the fact that corporate failures are usually expensive in terms of the cost of employment, lost output, and banking sector distress in cases where the corporate sector is heavily dependent on bank financing. Moreover, to the extent that the banking sector enjoys full or partial financial guarantee from the government, the losses incurred by the banking sector could translate into a higher debt burden for the public sector as the government issues bonds to cover the cost of recapitalizing banks. In cases where the risk transmission among sectors is magnified because of the capital structure linkages between sectors, problems that appeared isolated in the corporate sector could have far reaching consequences, triggering severe economy-wide financial crises—what we refer to as “macrofinancial” risk. The cost of corporate failures tends to be more acute in emerging markets than in mature markets because corporate financing is less diversified, is more vulnerable to sudden capital outflows and sharp changes in both world interest rates and the exchange rate, and there are fewer avenues to hedge or absorb financial losses. Developing an effective approach to detect corporate vulnerabilities before they become severe is essential in minimizing macrofinancial risks, thereby protecting the stability of the financial system and overall economy.

The contingent claims approach was developed from modern finance theory and has been widely applied by financial market participants to measure the default probability of a firm based on the market prices of the firm’s debt and equity. In this paper we apply the contingent claims approach (CCA) on an aggregated level to estimate corporate sector credit risk and evaluate the potential costs of macrofinancial risk transfers. In particular, we examine the ability of CCA to estimate probability of default within the corporate sector, assess and value the potential for risk transfer, and serve as an early warning indicator.

Since market prices represent the collective views and forecasts of many investors, the contingent claims methodology is forward looking—unlike analysis based only on a review of past financial statements—and helps increase the predictive power of the estimates of default risk. The ability to translate continuously adjusting financial market price information into current market value estimates of asset value is especially important given the speed with which

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2 The Asian crisis during the 1990s is often cited as an example where widespread bankruptcies of highly leveraged companies contributed to sharp declines in output and significant losses in the banking sector. Eventually, the public sector was forced to recapitalize the banking sector at a significant fiscal cost.

3 See Crouhy, Galai, and Mark (2000), Merton (1998). Perhaps the most widespread application of CCA has come from Moody’s KMV (MKMV) whose products are used by more than 2,000 leading financial institutions and firms in over 80 countries. Utilizing 30 years of historical data over 6,000 public and 70,000 private company default events for a total of 70,000 public and 1 million private companies, healthy and distressed, around the world. MKMV uses firm asset value, future asset distribution, asset volatility and the level of the default barrier to derive the firm specific probability of default, which they refer to as the Expected Default Frequency™ (EDF).
economic conditions change relative to the time span between releases of consolidated accounting balance sheet information. In contrast, accounting based approaches to corporate credit risk rely on historical balance sheet information which arrives with a significant lag, usually 90 days after the quarter or annual ending period. Furthermore, CCA takes into account the volatility of assets when estimating default risk. The volatility of assets are crucial in this process since firms may have similar levels of equity and debt, but very different probability of default if underlying asset volatility differs.

We use CCA to estimate a range of default risk indicators at the aggregated industry level for the non-financial corporate sector, including distance to distress and probability of default. This is the first and central purpose of the paper. However, solely focusing on the corporate sector alone ignores important possibilities for risk transfer across the consolidated balance sheets of the corporate, financial, and public sectors. Therefore, the second purpose of this paper is to extend the contingent claim methodology to a multi-sector framework, where linkages between the corporate, financial and public sectors can be examined. A multi-sector analysis allows for a more thorough understanding of the potential feedback effects between sectors and implications, if any, for the health of the financial and non-financial corporate sectors.

Through two case studies, Brazil and Thailand, we apply the CCA using the Moody’s Macro Financial Risk (MfRisk) model to assess the potential value of such models to act as early warning indicators of vulnerability. We apply the framework to retroactively assess the likelihood of corporate failure at the industry level and compare the results with actual events. We then extend the framework to a multi-sector analysis to estimate the level of risk transmission at the macro level between the corporate, banking and public sectors. We conclude with a discussion of the results, including advantages and disadvantages of the contingent claims methodology and potential measures to mitigate risk through enhanced surveillance and policy design.

II. CONTINGENT CLAIMS ANALYSIS

Initial theoretical work on contingent claims focused both on the pricing of options and the application of option theory to the analysis of the corporate capital structure. Since the total

4 The accounting based approach maps a reduced set of financial accounting variables to a risk scale to discriminate between repayment and non-repayment. A prominent accounting based approach was developed by Altman (1968) and used a linear combination of five accounting and market variables to produce a credit score – the so-called “Z-score”. A subsequent seven factor “Zeta model” was later introduced by Altman et al. (1977).

5 The MfRisk model was developed under a joint research effort between Moody’s and Macro Financial Risk, Inc. Access to MfRisk is only available through subscription.

6 Black-Scholes (1973) and Merton (1973, 1974) use no-arbitrage principles to derive the theoretical valuation formula for options commonly known as the “Black-Scholes Option Pricing Model”. The authors also discuss the application of option pricing to corporate liabilities. In subsequent work, Geske (1979) shows how options on equity are really compound options on firm value and expands the Black-Scholes formula to cover such cases.
value of the firm is equal to the sum of the value of the securities in the capital structure, the securities in the capital structure can be viewed as contingent claims on the underlying value of the firm.\textsuperscript{7} CCA can be used to analyze how the value of the contingent claim changes as the value of the firm changes through time. Therefore, contingent claims analysis should be viewed as a generalization of option pricing theory with the aim of specifying a framework within which all contingent claims can be valued.

Contingent claims analysis is based on three simple principles: (i) the value of liabilities flows from assets, (ii) liabilities have different seniority (and thus have different risks related to their seniority), and (iii) there is a random element to the way asset value evolves over time. Debt is a senior claim on the asset value and equity has a junior or residual claim on the asset value. Debt is risky because asset value may not be sufficient to meet the promised debt payments. The value of risky debt, therefore, can be seen as having two components, the \textit{default-free value} of the debt (promised payment value) and the \textit{expected loss} associated with default when the assets are insufficient to meet the promised payments on the debt. The value of the junior claim (equity in the case of firms) is derived from the residual value after the promised debt payments have been made.

If the value of assets has a random component (e.g. price changes, shocks and other factors affect asset value), higher asset volatility means there is a greater probability that assets will fall below the level necessary to meet the senior debt payments over the horizon period. Consequently, higher volatility means higher expected loss and a lower value of risky debt, other things equal. Financial techniques, namely option pricing relationships, have been developed to measure the expected losses as a function of the asset value, asset volatility, the default free value of debt, and the time horizon. Similarly, the value of equity and junior claims can be measured as a function of the same variables. The expected loss in risky debt is an implicit put option. Equity and junior claims are implicit call options.\textsuperscript{8}

The essence of CCA is that changes in observed variables – the value of securities in the capital structure – are used to infer changes in unobserved variables – the value of the firm. The application of CCA to the capital structure derives from the seniority of liabilities in the capital structure and the balance sheet identity that the total market value of debt plus equity must equal the current market value of the firm.\textsuperscript{9} Since equity represent a residual claim with limited liability, there is a correspondence between equity and a call option. In other words, the

\textsuperscript{7} The general definition of a contingent claim is any asset whose future payoff is contingent on the outcome of an uncertain event. In this context, is the right to obtain the residual value of the asset or the obligation to receive (residual) value of an asset.

\textsuperscript{8} Government guarantees to banking and financial sector increase with the volatility of assets in that sector. The government guarantees are also implicit put options which are a function of the banking and financial sector asset value, the associated asset volatility, the associated deposits and debt obligations, and the time horizon.

\textsuperscript{9} Securities on the liability side of the corporate balance sheet could also include senior collateralized debt, convertible securities, and preferred equity.
equityholder can simply walk away if the value of the firm drops below the face value of debt. The payoff on debt at maturity is equivalent to the difference between the default-free value of the debt and a put option on firm value with the strike price equal to the book value of the debt and the expiration date equal to the maturity of the debt. The put option represents the limited liability claim written by the debtholder to the equityholder which gives the equityholder the freedom of surrendering firm assets and walking away when firm value falls short of the promised payment to the debtholder at maturity.

A. The CCA methodology

In this section we briefly illustrate the contingent claims methodology as applied to a simplified corporate balance sheet comprised of senior debt and junior equity. At any point in time, the total market value of assets, $A$, of a firm financed with debt, $D$, and equity, $E$, is equal to the market value of equity plus market value of risky debt. Fundamental analysis dictates that firm asset value is derived from the stochastic discounted present value of income minus expenditures with the potential for asset value to decline below the point where scheduled debt payments can be made. If assets fall to a level where debt cannot be services, then default is the result. This level is often referred to as distress barrier, $DB$, and is equal to or close to the default-free value of debt.11

Equityholders have a junior contingent claim on the residual value of assets in the future. In this manner, the value of equity can be viewed as an option where holders of equity receive the maximum of either assets minus the distress barrier, or nothing in the case of default. The value of equity, therefore, is,

$$E = \max[A - DB, 0].$$

The standard option pricing formulas can then be used to relate changes in the price of firm assets to changes in equity.12 Given the relationship between firm equity and firm assets, changes in the value of traded equity can be used via option pricing relationships to infer changes in the market value of firm assets.

The case of risky debt, however, is slightly more complex. Holders of debt are obligated to absorb losses in the event of default and the guarantee of repayment by the lender can be modeled as an implicit put option since debtholders receive assets of the defaulted firm (or equivalently, the assets of the firm get “put” to the debtholders). Thus, holders of risky debt receive the minimum of the default-free value or, in the event of default, the senior claim on

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10 See Appendix I for additional detail on contingent claims analysis and the application of the Black-Scholes option pricing formula.
11 Analysis by MKMV shows that from an empirical point of view the distress barrier is approximated by short-term debt plus interest on long-term debt plus one-half of long-term debt.
12 See Appendix I for details.
assets. Since the value of default-free debt is equal to the distress barrier and the implicit put option on the assets of the firm yields \( \max [ DB - A, 0 ] \), the market value of risky debt can be modeled as,

\[
D = \min [ A, DB ] = DB - \max [ DB - A, 0 ]. \tag{2}
\]

Inserting these option pricing relationships into the economic balance sheet identity results in a market value of firm assets at time \( t \) of,

\[
A = D + E, \tag{3}
\]

\[
A = DB - \max [ DB - A, 0 ] + \max [ A - DB, 0 ]. \tag{4}
\]

The option pricing formula is used in a two step process. First, the observed market value of equity and the distress barrier are used with the call option formula to derive the value of firm assets. The value of firm assets and the distress barrier are then used with the put option formula to derive the implied market value of risky debt. Thus, the CCA uses call and put option pricing formulas to develop a market value balance sheet based on observed financial market variables and financial statement information.

**B. Distance to Distress and Probability of Default**

The two most useful credit risk indicators that arise from the implementation of CCA are the distance to distress and probability of default. As discussed in more depth in Appendix I, the option pricing formulas applied in CCA rely on only a few select variables: the value of the underlying asset, volatility of the underlying asset, the distress barrier, the risk-free interest rate, and time. These variables can be combined into a single measure of default risk. This measure, called the distance to distress, compares the difference between the market value of the firm relative to the distress barrier and then scales this distance based on asset volatility. In the application of CCA to actual firm capital structures, most practitioners compute the distress barrier as the sum of the book value of total short-term debt and one-half of long-term debt plus interest on long-term debt. This computation is used since historical instances of firm defaults have shown that it is possible for the value of firm assets to trade below the book value of total debt for significant periods of time without a default if most of the debt is long-term. Short-term debt, however, is more binding since the firm faces rollover risk in a shorter period of time. Thus, an adjustment is made to reduce the weight of long-term debt in the distress barrier.

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13 Although not explicitly discussed in this paper, a third useful credit risk indicator that can be obtained from CCA is the credit spread, or the additional risk premium required by bondholders to compensate for expected loss. See Appendix I for additional details.
The distance to distress combines the difference between assets and distress barrier with the volatility of assets into one measure,

\[
\frac{(\text{Market value of assets} - \text{Distress barrier})}{(\text{Market value of assets})*\text{(Asset volatility)}}
\]

which yields the number of standard deviations of asset value from distress. The distance to distress for a hypothetical firm is illustrated in Figure 1. The numerator above measures the distance between the expected one-year ahead market value of firm assets and the distress barrier. This amount is then scaled by a one-standard deviation move in firm assets. Lower market value of assets, higher levels of leverage, and higher levels of asset volatility all serve to decrease the distance to distress.

The final step to the probability of default consists of a mapping between the distance to distress measure from the equation above and actual probabilities of default based on historical data.\(^{14}\) Using historical information of a large sample of firms and given a distance to distress, Moody’s KMV is able to estimate the proportion of these firms that actually defaulted in a one-year ahead time horizon.

C. Moody’s MfRisk Model: Contingent Claims Analysis in a Multi Sector Framework

While the CCA approach as applied to individual corporate balance sheets has become a useful and widely applied tool in risk analysis, the focus on the corporate sector alone is too narrow to fully assess vulnerabilities. The corporate, financial, and public sectors are linked and changes in value in one sector can transmit risk from one sector to another. Changes in the value of assets of one sector lead to changes in value of liabilities of that sector which in turn affect the value of assets and liabilities in other sectors. For example, a decline in the assets of the corporate sector leads to a decline in the value of risky corporate debt as the expected losses increase. The lower value of debt held by banks lowers bank assets and causes the value of bank liabilities to fall and increases the likelihood of banking sector financial difficulties. Since governments frequently provide explicit or implicit financial guarantees to banks and large financial institutions out of concerns of systemic risk, the guarantee to the financial sector is a contingent liability on the sovereign balance sheet and a contingent asset on the financial sector balance sheet. As recent history has shown, these guarantees can become very large – from 20 to 70 percent of GDP in recent emerging market crises. These guarantees can be modeled as implicit put options.\(^{15}\)

\(^{14}\) This step is necessary since the option pricing formula used in the derivation of the distance to distress generally overstates actual probability of default. The Black-Scholes option pricing formula results in risk-neutral probabilities of default since the formula is derived from no-arbitrage conditions. Moody’s KMV has demonstrated using historical instances of default that the actual probability distribution has fatter tails than the normal distribution applies. See Crosbie and Bohn (2003) and Jarrow and Turnbull (1997) for additional information.

\(^{15}\) See Merton (1977), for example.
The existence of these linkages via risky debt and guarantees allows for risk transfer across balance sheets, meaning corporate sector vulnerabilities may come from “inside” the business environment or from “outside” if the value of the guarantee were to change suddenly. Therefore, a complete analysis of corporate sector vulnerabilities and potential for risk transfer requires a set of interrelated balance sheets across the corporate, financial, and public sectors.\(^\text{16}\) The ability to recognize and price both the expected losses in corporate debt and implicit guarantees is essential for conducting a full assessment of corporate sector vulnerabilities.

The Moody’s M\(_f\)Risk model is a practical application of CCA to the major sectors of a national economy in order to measure and analyze macrofinancial risks.\(^\text{17}\) The model constructs market value balance sheets of assets, debt liabilities, and junior claims for the aggregated corporate, financial, and public sectors to measure risk exposures and analyze risk transmissions between sectors. Hence, the M\(_f\)Risk model builds upon previous CCA credit risk methods by analyzing the probability of private and public debt defaults, currency crises, convertibility risk, cross-balance sheet vulnerability, and the values of the government guarantees provided to the financial and/or corporate sectors. Like Moody’s KMV, Moody’s M\(_f\)Risk uses the same approach to calculate distance to distress, default probabilities, and other risk indicators.

### III. Assessing Corporate Sector Vulnerabilities

To assess the ability of contingent claims methods to predict corporate sector vulnerabilities, we apply the Moody’s M\(_f\)Risk model to calculate retroactively the distance to distress and estimated actual default probabilities of the non-financial corporate sector for two case studies: Thailand and Brazil.\(^\text{18}\) CCA can be applied at the sector level in one of two ways. The sector can be built up from individual firm CCA models (e.g. Moody’s KMV) or the principles of CCA can be applied to the aggregate balance sheet for each of the main industry sectors of the non-financial corporate sector. Under the latter, CCA treats each industry sector as if it were one large firm. A negative feature of aggregating across industry sector is that it may be possible for individual firm weakness to be masked in the aggregation process since the ability of any one firm to impact the industry market value balance sheet is dependent on firm size relative to the industry as a whole. However, the aggregation process should be indicative of possible systemic vulnerabilities since a critical mass of firms is needed to influence the overall industry sector balance sheet. Aggregation maintains the fundamental premise underlying the CCA approach: (i) that the value of sector liabilities flows from sector assets, (ii) the liabilities have different seniority, and (iii) there is a random element to the way sector asset value evolves over time.

\(^{16}\) Gray, Merton, and Bodie (2003); Gray (2002).

\(^{17}\) See Appendix II for additional information on the M\(_f\)Risk multi-sector model and consolidated balance sheets.

\(^{18}\) Unless otherwise indicated, all reported values from applying CCA to the country cases were obtained from the M\(_f\)Risk model.
The next sections describe the application of the Moody’s M/Risk model to the corporate sectors in Brazil and Thailand during 2002 and 1997, respectively. Historical industry balance sheet information was combined with actual market price information to compute distance to distress and probability of default. The resulting vulnerability indicators across industry sectors are then compared with the results of actual corporate sector defaults to test the ability of the aggregated contingent claims approach to serve as an early warning indicator of corporate sector vulnerabilities.

A. The Brazilian Corporate Sector

Demand for capital in Brazil during the 1990s was spurred by large-scale privatizations accomplished through the use of leverage and, in some cases, existing leverage was sold with the asset itself. Consequently, corporate sector leverage increased during this time in the form of dollar-denominated, short- and medium-term securities (Figure 2).19 The sectors that were most leveraged were transportation, metals and mining, food and beverage, pulp and paper, and vehicle and parts (Table 1). Official data indicated that the overall corporate sector had a sizable negative foreign exchange (FX) position on its balance sheet before hedging through derivative operations.20

Some industry sectors are naturally hedged since they have export proceeds. However, the export base of Brazil is fairly small – at the time, merchandise exports of goods and services amounted to about US$70 billion (15 percent of GDP) – in relation to total external debt.21 While FX debt ratios were high in the metals, mining, and pulp and paper sectors, these sectors have the ability to generate FX revenues and, consequently were better positioned to weather FX shocks. However, currency mismatches were likely present in sectors that operate primarily in the domestic economy, such as electric utilities, telecommunications, and retail trade. FX hedging was accomplished by the use of FX derivatives intermediated through the financial system and facilitated by the provision of FX-linked domestic public sector debt.22

19 Data for the corporate sector represents quoted companies on the São Paulo stock exchange (BOVESPA) from Economatica, a private data provider. The companies in the sample hold about seventy percent of the overall non-financial private sector external debt and includes some public sector companies, but Petrobras has been excluded.
20 Official data indicates the corporate sector had US$63 billion in assets at end-2001 against a debt level of about US$90 billion. Corporate sector assets included US$51 billion registered as direct investment abroad and were likely less liquid.
21 Total external debt to total exports of goods and non-factor services was about 300 percent.
22 FX hedging via derivatives is wide-spread and Brazilian firms hedged foreign exchange risk, mainly by the use of FX derivatives. The public sector provided the corporate sector with interest rate and currency protection through the highly indexed structure of public debt. Banks acted as intermediaries by buying the dollar-linked bonds or FX swaps issued by the public sector and then selling the FX hedge to the corporate sector. Firms also hedged themselves on the BM&F, the local derivative market, which is very active.
B. CCA and Financial Market Uncertainty in 2002

Applying the CCA framework retroactively to the aggregated industry sectors of publicly listed companies on the Brazil stock exchange suggests that this approach would have provided an accurate view of the pending financial difficulties within the Brazilian corporate sector. Using the historical balance sheet capital structures in conjunction with historical traded equity prices as inputs, the option pricing framework relates these to the value of industry assets. Prior to the financial market volatility, most industries of the corporate sector were two or more standard deviations away from the sector distress barrier, correlating to a relatively low probability of default over a one-year ahead time horizon (Figure 3). During the second half of 2002, fears over policy continuity following the upcoming presidential elections combined with weakening sentiment in external capital markets caused a deterioration in financial market conditions in Brazil. Access to international capital markets by both the public and private sector came to a halt beginning in July of 2002 and pressure on the currency intensified. The real depreciated from R$2.3 per U.S. dollar in April 2002 to nearly R$4.0 in October before ending the year at R$3.5.

Equity valuations began to decline in April of 2002 and did so on a near-continuous basis before bottoming out in October. As equity valuations declined significantly, the distance to distress decreased for many firms and resulting probabilities of default were at their highest at end-September 2002 (Figure 4) when financial uncertainties peaked. Despite the high volatility of asset prices and capital outflows, equity markets remained selective. In particular, while the ensuing market uncertainty affected all industries, equity price declines were more concentrated in industries that had operations primarily in the domestic economy as opposed to export led sectors. Industries that operated in the domestic economy; such as retail trade, textiles, home appliances (electronics), utilities, and food and beverage; were more likely to have dollar liabilities on their balance sheets versus local currency revenue streams. At the height of the crisis, the distance to distress for these industries ranged from (0.3) standard deviations for home appliances to 0.4 standard deviations for chemicals, indicating high levels of balance sheet distress. In contrast, the main export industries; mining, oil and gas, steel, pulp and paper, and petroleum chemicals; were less affected since currency mismatches between assets and liabilities were less prevalent. Distance to distress for these sectors averaged 1.1 standard deviations, well below their levels in March, but well above the distance to distress of their non-export based counterparts. Therefore, markets discriminated against non-export sectors more heavily when weighing the impact of the depreciation and economic slowdown.

The electric utility industry is an example of an industry that operates primarily in the local market, with revenues in local currency and some liabilities in foreign currency. The electric utility sector raised large amounts of funds during 1998-99 in dollar loans largely related to privatization efforts. The subsequent devaluations in the currency in 1999 and 2002 along with the rationing of power in 2001 resulted in balance sheet weaknesses. As shown in Figure 5, the

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23 Private sector credit risk analysts and Moody’s MfRisk associates view a one-year ahead distance to distress measure below 0.5 as an indication of severe stress.
distance to distress in the utility sector narrowed steadily and reached its lowest level at end-September. Prior to the financial market volatility, the utility sector had a distance to distress of slightly above 2 standard deviations. The subsequent mapping of this distance to distress into probability of default indicated that the aggregated industry had a one-year ahead probability of default equal to 5 percent (Figure 6). In September, when equity price declines were largest and implied asset volatility had reached its peak (Figure 7), the distance to distress for the sector had fallen to a standard deviation of 0.2 at its lowest recorded level and averaged a standard deviation of 0.6 for the entire month. This average standard deviation was equivalent to a one-year ahead probability of default of around 30 percent for the sector as a whole.

The highest profile case of distress within the electric utility sector was Eletropaulo, Latin America’s largest power distributor and a subsidiary of AES. On August 26, 2002, Eletropaulo was placed in selective default by Standard & Poor’s after missing a debt payment and approached holders of R$700 million (US$223 million) in debt with a plan to extend payments by two years. Other instances of weakness in the utility sector were partially resolved by an infusion of government lending or capital infusion by the parent company. In addition to the utility sector, there were other selected high profile cases of corporate defaults that confirm the level of distress indicated in the MfRisk model output. For example, Globopar, Latin America’s largest media company, defaulted on US$1.5 billion worth of debt in October 2002. This decision also pushed one of its subsidiaries, Globo Cabo S.A., into default on nearly US$100 million of debt in early November. Varig, the largest airline carrier in Latin America, agreed with creditors in September to reschedule debt payments after returning some of its leased aircraft and renegotiating more favorable terms on remaining leases. Finally, BCP Telecommunications, the wireless unit of BellSouth and Safra Group, defaulted on a US$375 million loan in late March.

Outside of some systemic weakness in utilities and media-telecommunications, the Brazilian corporate sector weathered the financial volatility in 2002 relatively well due to, in part, the rapid improvement in the economic environment after the election-related uncertainties passed. There were instances of capital infusion from parent entities or public sector sources, but this process was more the exception rather than the rule and widespread corporate defaults were avoided. In contrast to the Brazil case study, the Asian crisis in 1997-1998 is often cited as an example of bankruptcies of highly leveraged companies that eventually posed systemic vulnerabilities to the financial and public sector. The next section uses the MfRisk model to retroactively assess the aggregated CCA on the Thai corporate sector during the Asian crisis.

C. The Thai Corporate Sector

Strong economic growth, along with a fixed nominal exchange rate and capital account liberalization, contributed to a surge in capital inflows in Thailand and most of South East Asia from 1992-1996. Private sector investment in manufacturing and real estate was complemented by public sector investment, especially in infrastructure relating to transportation, telecommunications, and utilities. Private sector capital inflows into the five most affected
economies from the Asian financial crisis increased from US$30 billion in 1992 to US$73 billion in 1996. Unlike in Latin America, the bulk of the capital inflows into Asia came in the form of bank lending, which accounted for US$120 billion out of the total US$234 billion (51 percent) of private capital flows.

Much of the bank lending went to the non-financial private sector in the form of short-term U.S. dollar denominated loans. About 65 percent of all external borrowing was conducted by the non-financial corporate sector from 1992-1996 (Table 2) and the majority of this debt was contracted at short maturities. Nearly 75 percent of all short-term debt had original maturities of less than six months, with 1-3 months being the most popular time horizon (Table 3). Successive years of a heavy reliance on debt relative to equity financing resulted in a debt-to-equity ratio of nearly 160 percent in 1996, the highest in Asia at that time. Even with increasing quantities of short-term U.S. dollar denominated debt on corporate sector balance sheets, the widespread perception that the peg would remain stable contributed to low hedging positions. While data on overall corporate sector hedging practices are unavailable, studies are available that provide insight into hedging behavior. For example, one study examined 29 large non-financial firms and found that 85 percent of the total foreign debt positions were unhedged.

The real estate and asset price bubble came under pressure beginning in 1996 as growth began to slow significantly in the second half of the year. Successive years of capital inflows and an appreciating US dollar after 1994 caused an appreciation in the real exchange rate. A higher real exchange rate combined with decreasing external demand led to a sharp reduction in export growth from 25 percent in 1995 to (2.0) percent in 1996. Manufacturing output, especially in the areas of durable goods and associated inputs, fell sharply. Declines were also registered in beverages and textiles. Overall, measures of capacity utilization in the manufacturing sector declined from 80 percent in 1995 to 76 percent in 1996.

D. CCA and the Asian Financial Crisis

While the generalized slowdown should have signaled an increase in the probability of an economic adjustment from built-up imbalances, traditional signals of vulnerability did not

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24 IMF (1998). The five economies are Indonesia, Korea, Malaysia, the Philippines, and Thailand. Capital flows are measured as net foreign direct investment plus net portfolio investment plus net other investment.
26 SBC Warburg Dillion Read (1997). The level reported is the weighted average of unhedged short-term and long-term debt and should only be viewed as an indicative measure of hedge since firm-by-firm statistics in the sample varied widely. The firms in the sample accounted for US$16 billion in U.S. dollar denominated debt, of which 15 percent was classified as short-term.
register much forewarning. However, aggregated CCA provided some indication of weakness prior to the severe financial market volatility that ensued. Equity prices began to decline in early 1996, nearly 18 months before the floating of the Baht, decreasing the distance to distress for many firms. Since much of the accumulated debt during previous years was short-term and dollar denominated, the distress barrier became more binding since firms were vulnerable to both rollover and exchange rate risk. By the end of 1996, the distance to distress had fallen in three of the four main non-financial industry sectors. Manufacturing, trade and services, and real estate and construction, all registered lower distance to distress relative to their 1992 positions while agriculture registered a slight increase. Industry level assets relative to distress barriers in 1992 and 1996 are displayed in Figure 8. The distance to distress was lowest in the real estate and construction sectors, reflecting the increasing likelihood of a boom-bust cycle.

Serious FX market pressures began in early 1997 with the release of poor fiscal and export data. Concerns over increased monetization, a deteriorating current account, and non-performing assets in the financial sector caused foreign investors to begin unwinding carry-trade positions. The default by Samprasong Land in February and rising short-term interest rates in developed economies further fueled capital flight and FX pressures. Extensive intervention by the Bank of Thailand in support of the currency and the imposition of capital controls in May failed to stem capital flight and speculative positions by market participants. The baht was subsequently floated on July 2, 1997.

By the end of July, the baht had depreciated nearly 24 percent and equity prices 20 percent relative to their end-1996 levels. The depreciation substantially raised distress barriers for firms with dollar-denominated liabilities and equity prices continued to reflect a worsening economic outlook and capital outflows. By the end of July, distance to distress indicators showed a disturbing picture for the real estate and construction sectors, and substantial strains on the manufacturing, trade, and service sectors. As shown in Figure 9, standard deviations from the distress barrier were significantly negative for real estate and construction. Manufacturing, trade, and services were all within one-half of one standard deviation from distress.

Following the initial turbulence, Thailand agreed on a US$17 billion stabilization program with the IMF. However, the currency continued to depreciate and equity prices continued to fall. By the end of October the baht had fallen 58 percent from 25 to 40 B/US$ and the SET Index stood at 447, or 46 percent below its end-1996 value. At these levels, CCA implies a very serious crisis with widespread corporate defaults as distance to distress becomes negative for all

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28 Radelet and Sachs (1998) examine a wide variety of macrofinancial indicators for the Asian crisis countries. According to their analysis, indicators were highest in Thailand and Korea, but alarms were few in Indonesia, Malaysia, and the Philippines.
29 Recall that the distress barrier is the sum of the book value of short-term debt plus one-half of long-term debt and interest. Higher ratios of short-term debt to total debt imply a more stringent barrier.
30 A carry trade is a short-term position put in place to take advantage of persistent interest rate differentials and a fixed-exchange rate. See IMF (1998, p. 44) for additional details.
31 International Monetary Fund (1998).
corporate sectors except agriculture (Figure 10). The CCA approach effectively predicted widespread corporate defaults after the floating of the baht. According to reports by financial institutions to the Bank of Thailand, around US$5 billion of corporate debt, or 3 percent of GDP, was engaged in restructuring in June of 1998. This amount rose to around US$19 billion, or 16 percent of GDP, in June 1999. Of this total, an estimated two-thirds was held by domestic commercial banks with the balance evenly split between foreign banks and finance companies.

IV. MULTI-SECTOR CONTINGENT CLAIMS ANALYSIS

While the examples of the Brazilian and Thailand corporate sectors highlights the usefulness of CCA as a tool in risk analysis, the focus on the non-financial corporate sector alone may be too narrow to fully assess vulnerabilities. As previously indicated, third party guarantees like implicit government guarantees of the financial system are present or implicit across the balance sheets of the public sector, financial sector, and non-financial corporate sector. Therefore, a complete analysis of corporate sector vulnerabilities and potential for macrofinancial risk transfer requires a set of interrelated balance sheets across the corporate, financial, and public sectors (the composition of the consolidated balance sheets is detailed in Appendix II with special emphasis on application of CCA to the public sector). The following sections discuss two examples of application of CCA to a multi-sector framework. First, the Brazil example illustrates risk transfer from the public sector to the corporate sector balance sheet. Second, the Thailand example illustrates risk transfer from the corporate sector to the balance sheets of the financial and public sectors.

A. Multi-Sector CCA – Brazil

In the context of Brazil, the multi-sector CCA framework can be used analyze risk transmission during 2002 between the public sector and the corporate sector through explicit guarantees since the public sector has played an active role in providing interest rate and currency hedges to the corporate sector. These hedges were provided through the regular issuance of domestic public sector debt with returns linked either to short-term interest rates or the real/US$ exchange rate. The financial market instability during 2002 caused some market participants to call into question the sustainability of the public sector debt and, in turn, the value of the interest rate and currency hedges provided to the corporate sector.32 Through the provision of an explicit guarantee, any severe public sector distress is transmitted to the asset side of the balance sheet of the corporate sector.33

32 Spreads on external Brazilian public sector debt tripled to over 2400 before rallying to 1400 at year-end. The rollover rate on domestic public debt fell below 50 percent and average maturity of newly issued debt fell to 12 months, causing a decline in the average maturity of the stock of domestic public debt from 36 to 32 months.

33 To the extent that the corporate sector holds government debt directly, changes in the value of these assets would appear on the balance sheet. As discussed below, currency hedge is typically intermediated through the financial system and would constitute and off-balance sheet item.
The liabilities of the public sector includes external debt, domestic debt, financial guarantees, and base money. In addition to liquidity and role as a medium of exchange, domestic money is also valued since it can be exchanged for foreign currency. As domestic currency is exchanged for foreign currency, FX assets of the public sector decline. If sovereign external debt payments have first claim on foreign currency assets in the public sector balance sheet, then only residual foreign currency reserves in excess of scheduled external debt payments are available for conversion from domestic to foreign currency holdings. Therefore, the holders of domestic currency have a claim on the residual foreign currency assets of the public sector above what is necessary to service public sector external debt.

This claim can be viewed as a call option on government foreign currency assets. The payoff is equal to the maximum of residual FX reserves or zero if, (i) FX reserves fall below external debt service costs, or (ii) capital controls are put in place to restrict convertibility. Restrictions on the ability to convert local currency to foreign currency would prohibit the ability of the corporate sector to service external debt. CCA can be applied to determine the probability of an “inconvertible” event which is an important measure of corporate sector vulnerability. If capital controls are not put in place and reserves continue to decline, then government solvency may be called into question as government assets approach the distress barrier.

At end-2001, Brazil had gross international reserves of US$36 billion and external debt of the non-financial public sector amounted to US$93 billion. External debt service costs in 2002 were estimated at US$7.4 billion of interest and US$6.5 billion of principal. The public sector also had outstanding domestic dollar-linked debt and FX swaps of US$65 billion at the end of 2001, much of which was short-term. However, these instruments are denominated and payable in domestic currency, easing the government’s foreign currency constraint. Reserve coverage, therefore, appeared adequate to cover one-year ahead hard foreign currency requirements of the public sector.

In contrast, foreign currency liabilities of the private sector were significant. At end-2001, total external debt of the financial and non-financial corporate sector amounted to US$117 billion, much of which was short-term. External debt service costs for the private sector were estimated at US$8 billion in interest and US$30 billion of principal. The private sector hedged this FX exposure through the government issued dollar-linked debt and FX swaps, intermediated through the financial system. While denominating these instruments in the local currency eases the government’s foreign exchange constraint, it does not ease the hard currency constraint on the private sector. The hedge position is a domestic currency asset of the private

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34 Convertibility risk can also be measured through the differential between the NDF offshore contract and the implied onshore FX forward.

35 Risk transmission from the corporate sector to the banking sector is relatively mild. Private sector credit as a percent of GDP is only 25 percent and FX exposure is only intermediated from the public sector to the private sector. Therefore, net FX exposure remains on the public sector balance sheet as opposed to the banking sector.
sector versus foreign currency liabilities. The holders of the hedge must still convert *reais* into foreign currency, mainly U.S. dollars, to complete the transaction.

During the currency volatility in 2002, CCA indicates that market perception of convertibility risk became sizable. As access to international capital markets dried up and rollover rates on external debt fell, demand for dollars increased and public sector international reserves fell. The result was an increased perception of convertibility risk as a subset of public sector assets (international reserves) approached an implicit distress barrier (one-year ahead foreign currency liabilities of the public sector). Local markets were not seen as providing an effective hedge partly because all local contracts were settled in the domestic currency. As shown in Figure 11, the probability that restrictions would be placed on currency convertibility rose from 10 percent in March to 50 percent in September.\(^{36}\) As the perception of convertibility risks increased, firms either stopped rolling over FX hedge or even sold existing hedge in order purchase dollars in the spot market.\(^{37}\) This withdrawal of hedge coincided with prepayments and capital outflows of nearly US$7 billion during the same time period.\(^{38}\) As uncertainties subsided, both convertibility risk and capital outflows declined rapidly by year-end. Overall, the use of CCA to estimate convertibility risk appears to explain some of the rationale behind the capital outflows and corporate sector behavior during the second half of 2002.

The Brazilian government took many steps to address the market uncertainty, focusing on a set of core policies that maintained discipline and restored market confidence. The government increased the primary surplus and tax reform and pension reform were given priority in the reform agenda. Consistent policy implementation and improving fundamentals eventually led to a normalization of financial markets. For example, nine months after external access was halted and six months following the peak of financial market turbulence, the sovereign re-accessed external capital markets in April 2003. Widespread access by the corporate sector to external markets returned shortly thereafter.

### B. Multi-Sector CCA – Thailand

In the context of Thailand and the Asian crisis, the multi-sector CCA framework can be used to analyze risk transmission from the corporate sector to the financial sector to the public sector. The size of the risk transfer across the balance sheets is captured in two steps. First, equity of the financial sector is modeled as a call option on total financial sector assets, yielding estimates of changes in the market value of financial sector assets over time. Second, the financial guarantee from the public sector is modeled a put option based on the derived market value of

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36 The convertibility risk measure in Figure 14 is displayed as a monthly average.

37 The notional value of registered outstanding hedge positions registered on the CETIP, a securities clearing registry, declined from US$50 billion as of end-May 2002 to US$37 billion at end-October. Amounts on CETIP include financial companies and trading within own accounts and serves more as an indication of hedging practices as opposed to actual amounts.

38 Capital outflows are registered flows through so-called CC5 accounts and were obtained from the Central Bank of Brazil.
financial sector assets. The value of the put option also requires an assumption over recovery rates, which are normally less than full. For this analysis, a recovery rate of 80 percent was assumed for the banking sector and 60 percent for the non-bank financing companies. Based on this structure, declines in the value of financial sector equity from the depreciation of the baht and increases in loan delinquency rates cause the market value of assets to decline, increasing probability of default. As the probability of default rises, the value of the government guarantee adjusted for recovery rates increases.

As discussed above and displayed in Figures 9 and 10, distance to distress indicators for the industry sectors were low or negative by end-July 1997, especially in real estate and construction. By the end of October, corporate sector distance to distress measures were negative for all sectors except for agriculture, implying widespread corporate defaults. Corporate sector weakness was transferred to the balance sheet of the financial sector, reflected in lower equity prices, increased asset volatility, and decreasing distance to distress. As shown in Figure 12, distance to distress for the financing companies was already significantly negative in July 1997. In August 1997, around the same time Thailand agreed on the stabilization program with the IMF, the Bank of Thailand suspended operations in 42 financial companies. By October, the distance to distress for both commercial banks and financial companies were negative. The problems in the balance sheets of the financial companies became so severe that the Bank of Thailand closed 56 finance companies in December 1997.

The decreasing market value of financial sector assets relative to the distress barrier increased the value of the financial sector guarantee. Figure 13 plots the evolution of the one-year ahead estimate of the value of the financial sector guarantee throughout 1997. The value is listed as a percent of 1996 nominal GDP for scale purposes. As shown in the figure, the value of the guarantee was relatively small during early 1997, but had already increased to nearly 10 percent of GDP prior to the float, representing the building vulnerabilities on the balance sheets of the financial companies in particular. After the floating of the baht in July and subsequent turbulence, CCA estimated the value of the financial sector guarantee at between 30 and 40 percent of GDP by October 1997.

Subsequent restructurings in the financial sector indicate that the one-year ahead estimated value of the financial sector guarantee was relatively accurate. In October 1998, reports by private sector specialists estimated that 45 percent of loans were non-performing and that loan losses amounted to 27 percent of GDP or 350 percent of financial sector capital.39 The slow pace of the restructuring process meant that actual number of restructured loans was not known until several years later. Fortunately, the Bank of Thailand has published extensive data on the debt restructuring carried out by financial institutions after the 1997 financial market crisis.40 The database includes number of cases and amounts restructured in total and by type of loan.

Based on this data, financial sector debt equivalent to 23 percent of GDP had completed the

40 See the website of the Bank of Thailand and Dasri (2000) for additional information.
restructuring process by end-1999. By end-2000, this value had reached 40 percent of GDP. Both numbers roughly correspond to model estimates.

V. CONCLUSIONS

In this paper, we have examined the application of the contingent claims approach to identifying corporate sector and macrofinancial vulnerabilities. In doing so, we applied the Moody’s MfRisk model, which uses aggregated CCA principles, to assess vulnerabilities retroactively in the corporate sector as well as in a multi-sector setting through two historical cases. The results presented here indicate that the method holds promise in identifying corporate sector vulnerabilities and estimating the associated value of macrofinancial risk transfer across interrelated balance sheets of the corporate, financial, and public sector.

The main output of CCA is an estimated probability of default which is a function of the capital structure of the balance sheet, the volatility of asset returns, and the current asset value. Since the information contained in the firm balance sheet and equity price can be translated into a probability of default, CCA has underpinned the development of credit risk and risk management techniques. In particular, CCA is used as by: (i) major credit rating agencies to monitor and assign credit ratings, (ii) financial institutions to inform interest rate pricing on loans and set adequate levels of regulatory capital, and (iii) investment banks and insurance companies to assess value-at-risk. For example, when applied across a portfolio of firms, the probability of default multiplied by weighting within the portfolio creates a value-at-risk (VaR) indicator which is then used in conjunction with other VaR indicators to adjust capital adequacy or allow the firm to offset risk exposure by entering into offsetting financial transactions.

While CCA has been most often applied to individual firm balance sheets, this paper has used the Moody’s MfRisk model to demonstrate how CCA also applies to aggregated sector balance sheets or across balance sheets in a multi-sector framework. The evolution of CCA to a multi-sector framework incorporates the potential for risk transfer across balance sheets, providing an additional avenue for vulnerability analysis. The conclusions presented here show that such analysis is warranted in the event of: (i) a full or partial financial guarantee from the government to the financial sector, (ii) government provision of guarantees such as interest rate and currency hedges to the corporate sector, and (iii) because of the presence of a high degree of correlation across firm balance sheets (the chaebol structure in Korea is an example of the potential for increased interconnectivity across firm balance sheets).

A. Advantages and Disadvantages of CCA

The advantages of the contingent claims methodology are numerous, only some of which are discussed here. The main advantage of CCA is that it uses observable balance sheet and financial market data along with volatility to construct a measure of default risk. The ability to translate continuously adjusting financial market price information into current market value estimates of asset value is especially important given the speed with which economic conditions change relative to the time span between releases of consolidated accounting balance sheet information. Furthermore, balance sheet information arrives with a significant lag, usually 90
days after the quarter or annual ending period. CCA combines the liability structure of the balance sheet with present day information from financial markets to construct a market value estimate of the current balance sheet along with forward looking indicators of vulnerability. In addition, CCA distinguishes itself from other vulnerability analysis by recognizing the important role that volatility has in determining default probabilities. Increases in volatility increase the option value and benefits equityholders at the expense of bondholders. By capturing volatility, CCA accounts for the fact that firms with same capital structures may have different distance to distress and default probabilities.

While not explicitly detailed in this paper, the contingent claims methodology incorporates non-linearity which yield significant improvements in vulnerability analysis over traditional linear relationships. In option pricing theory, the value of the option is dependent on changes in the underlying asset. The rate of change of the price of the option relative to changes in the underlying asset is referred to as the delta. The equivalent measure in bond pricing is duration – how much the bond price changes in response to a change in interest rates. However, the duration measure is only accurate for small changes in the interest rate. In fact, duration changes as the interest rate changes and this measure is referred to as convexity. The same is true with option prices. The option delta is only accurate over small changes in asset price. The equivalent measure to convexity in option pricing is called the option gamma, or the rate at which the option delta changes as the price of the underlying asset changes. Therefore, the non-linearity of the Black-Scholes methodology allows for a more accurate description of changes in vulnerabilities from large changes in asset prices just as both duration and convexity are needed to compute accurate changes in bond prices from large movements in interest rates.  

There are several disadvantages of the CCA methodology, but most can be mitigated. First, the CCA uses on-balance sheet information to construct the capital structure of the balance sheet and off-balance sheet items like hedging or derivative positions may not be included. This fact, however, is not a disadvantage of CCA alone – it is a shortcoming of all vulnerability indicators that rely on balance sheet information. Rather, CCA relies heavily on financial markets to synthesize the current economic situation and its potential impact on the balance sheet and reflect this in current equity or junior claim prices. Thus, with respect to off-balance sheet positions, CCA is implicitly assuming that financial markets are the best source for information regarding the current value of a firm. This is not an unrealistic assumption since industry analysts, equity specialists, and other local market participants are likely in the best position to assess current trends and developments. However, CCA does not assume that market forecasts are always correct. Financial markets can be taken by surprise and the degree to which this happens limits the ability of CCA to accurately forecast vulnerabilities.

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41 Advances in option pricing methodology also include the option vega which can allow for changing volatility of the underlying asset. This feature is important since volatility of assets often increases significantly during periods of financial stress. Traditional linear relationships are, therefore, significantly inferior to their non-linear counterparts.
The CCA option pricing relationships derive a risk-neutral default probability based on cumulative normal distributions which must be mapped into actual expected default probability. The use of risk-neutral default probabilities generally overstates the actual probability of default, requiring a mapping to estimated actual probability of default based on historical data. Fortunately, Moody’s KMV has been able to use historical observations of public and private company defaults to map risk-neutral default probabilities into actual expected default probabilities. The resulting distribution has slightly fatter tails than a cumulative normal distribution. The mapping, however, is mainly representative of U.S. firms, although it has been successfully used in a wide variety of applications. Some emerging markets have poor quality information and many unlisted firms. Moody's has tried to address this problem with a blended approach using an option model and financial data (Riskcalc). Comparative studies have shown that the CCA-related models are much more accurate than Z-score accounting ratio models or accounting ratios in predicting default (around 70% accuracy ratios for the CCA type models and about 50 to 56% for the Z-score and accounting ratio models).\footnote{Sobehart and Stein (2000).}

Finally, application of CCA to the financial sector and public sector balance sheets is still a relatively new phenomenon. Credit risk within financial institutions is often difficult to disentangle given the inherent problems in analyzing and pricing illiquid loan portfolios and understanding true liability positions. However, equity markets understand these issues well and results from this analysis and Moody’s KMV suggest that CCA does well in assessing vulnerabilities within the financial sector. With respect to applying CCA to the public sector balance sheet, occurrences of sovereign defaults are small in number, making testing and calibrating of the model on sovereign balance sheets difficult. Moody's-MfRisk has overcome this problem by correlating the public sector risk indicators with credit spreads instead of a pool of defaults, allowing for calculation of sovereign spreads and default probabilities for a variety of different scenarios. Therefore, while application of CCA to the public sector is new, early results indicate that this application holds promise.

**B. Implications for Macroeconomic Risk Management**

At a minimum, application of CCA in a multi-sector setting allows for the ability to value the potential for risk-transfers across balance sheets and, in particular, the ability to value the public sector guarantee to the financial sector as shown in the case of Thailand. However, just as CCA is used by the private sector to analyze risk and implement risk management strategies, CCA applied in a multi-sector framework provides policymakers with the same ability at the public sector level. The multi-sector setting in the Moody’s MfRisk model provides a interconnected framework within which policymakers can analyze potential policy mixes and evaluate which may be more adept at countering vulnerabilities.

Policymakers can follow several strategies in order to transfer or mitigate risk, including: (i) a direct change in the balance sheet through policy action (e.g., increasing the primary surplus...
increases the assets of the public sector relative to the distress barrier), (ii) managing implicit or explicit guarantees (e.g., providing or withdrawing credit guarantees to the banking sector or currency hedge to the corporate sector), (iii) risk transfer strategies (e.g., signing contracts with foreign banks or insurance companies), (iv) institutional changes in markets (e.g., capital requirements, Basel Core Principles, payments systems, mark-to-market requirements), and (v) mitigating risk by diversification, hedging or insurance. Any of these strategies, either individually or in a combination can be analyzed in a multi-sector framework to manage risk in a dynamic economy. While not the subject of this paper, we suggest it as a useful avenue for further study.
References


### Table 1.
Brazil: Non-financial Companies, Leverage and Debt Structure, December 2001

<table>
<thead>
<tr>
<th>Sector</th>
<th>Debt/Equity</th>
<th>Liabilities/Equity</th>
<th>Current Liab/Liabilities</th>
<th>ST Debt/Total Debt</th>
<th>FX Debt/Total Debt</th>
<th>ST FX Debt/ST Debt</th>
<th>LT FX Debt/LT Debt</th>
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</thead>
<tbody>
<tr>
<td>Basic &amp; Fab Metal</td>
<td>1.50</td>
<td>2.44</td>
<td>0.39</td>
<td>0.43</td>
<td>0.76</td>
<td>0.76</td>
<td>0.75</td>
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<tr>
<td>Chemical</td>
<td>0.75</td>
<td>1.74</td>
<td>0.48</td>
<td>0.32</td>
<td>0.62</td>
<td>0.81</td>
<td>0.52</td>
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<tr>
<td>Construction</td>
<td>0.64</td>
<td>1.84</td>
<td>0.42</td>
<td>0.66</td>
<td>0.15</td>
<td>0.00</td>
<td>0.44</td>
</tr>
<tr>
<td>Electric Electron</td>
<td>0.75</td>
<td>2.27</td>
<td>0.59</td>
<td>0.72</td>
<td>0.63</td>
<td>0.70</td>
<td>0.47</td>
</tr>
<tr>
<td>Electric Power</td>
<td>0.58</td>
<td>1.15</td>
<td>0.33</td>
<td>0.23</td>
<td>0.70</td>
<td>0.48</td>
<td>0.76</td>
</tr>
<tr>
<td>Food &amp; Beverage</td>
<td>1.34</td>
<td>2.39</td>
<td>0.50</td>
<td>0.48</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>Industrial Machin</td>
<td>0.61</td>
<td>1.35</td>
<td>0.70</td>
<td>0.68</td>
<td>0.61</td>
<td>0.75</td>
<td>0.34</td>
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<td>Mining</td>
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<td>0.32</td>
<td>0.93</td>
<td>0.87</td>
<td>0.96</td>
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<tr>
<td>Oil &amp; Gas</td>
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<td>0.60</td>
<td>0.52</td>
<td>0.63</td>
<td>0.89</td>
<td>0.35</td>
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<td>Pulp &amp; paper</td>
<td>1.09</td>
<td>1.47</td>
<td>0.43</td>
<td>0.42</td>
<td>0.78</td>
<td>1.76</td>
<td>0.07</td>
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<tr>
<td>Telecommunication</td>
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<td>Textile</td>
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<td>1.35</td>
<td>0.56</td>
<td>0.58</td>
<td>0.44</td>
<td>0.55</td>
<td>0.30</td>
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<tr>
<td>Trade</td>
<td>1.00</td>
<td>2.59</td>
<td>0.69</td>
<td>0.49</td>
<td>0.72</td>
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<td>Transport Serv</td>
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<td>-82.84</td>
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<td>0.21</td>
<td>0.29</td>
<td>0.13</td>
<td>0.33</td>
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<tr>
<td>Vehicle &amp; parts</td>
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<td>2.51</td>
<td>0.70</td>
<td>0.66</td>
<td>0.64</td>
<td>0.49</td>
<td>0.91</td>
</tr>
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</table>

Sources: Economatica; and IMF staff estimates.

1/ This value was the result of a large loss incurred on a small negative shareholders equity.
Table 2. Thailand: Gross External Borrowing by Nonbank Sector 1992 – 1996

<table>
<thead>
<tr>
<th>Year</th>
<th>Short-Term Total</th>
<th>Short-Term Non-bank financial institutions</th>
<th>Short-Term Non-financial sector</th>
<th>Short-Term Industry</th>
<th>Short-Term Food</th>
<th>Short-Term Trade</th>
<th>Short-Term Construction</th>
<th>Short-Term Trade</th>
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<tr>
<td>1992</td>
<td>14,870</td>
<td>4,632</td>
<td>10,238</td>
<td>151</td>
<td>519</td>
<td>10,029</td>
<td>2,025</td>
<td>3,832</td>
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<td>1993</td>
<td>17,730</td>
<td>4,807</td>
<td>12,923</td>
<td>199</td>
<td>199</td>
<td>14,757</td>
<td>2,199</td>
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<tr>
<td>1994</td>
<td>16,076</td>
<td>6,578</td>
<td>9,498</td>
<td>156</td>
<td>156</td>
<td>6,900</td>
<td>1,999</td>
<td>4,999</td>
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<tr>
<td>1995</td>
<td>18,816</td>
<td>6,160</td>
<td>11,916</td>
<td>194</td>
<td>194</td>
<td>6,588</td>
<td>1,944</td>
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<td>1996</td>
<td>18,061</td>
<td>5,690</td>
<td>11,492</td>
<td>195</td>
<td>195</td>
<td>6,149</td>
<td>1,845</td>
<td>4,394</td>
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</table>

Source: Reprinted from IMF (2002). Data provided by the Thai authorities.
1/ Includes borrowing from affiliates; excludes commercial banks and BIBFs.
2/ Excludes real estate.

Table 3. Thailand: Borrowing Terms of Private External Loans, 1992 – 1996

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount of Borrowing</th>
<th>Amount of Borrowing</th>
<th>Amount of Borrowing</th>
<th>Amount of Borrowing</th>
<th>Amount of Borrowing</th>
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<tr>
<td></td>
<td>Millions $</td>
<td>Percent of Total</td>
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<td>Percent of Total</td>
<td>Millions $</td>
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<tr>
<td>1992</td>
<td>17,730</td>
<td>100.0</td>
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<td>1993</td>
<td>18,061</td>
<td>100.0</td>
<td>18,816</td>
<td>100.0</td>
<td>22,064</td>
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<td>1994</td>
<td>25,943</td>
<td>100.0</td>
<td>22,064</td>
<td>100.0</td>
<td>18,228</td>
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Source: Reprinted from IMF (2002). Data provided by the Thai authorities.
1/ Includes borrowings from affiliates; excludes commercial banks and BIBFs.
Figure 1. Distance to Distress

Figure 2. Brazil: Corporate Sector Leverage Indicators
Figure 3. Brazil: Distance to Distress by Sector in March 2002

Figure 4. Brazil: Distance to Distress by Sector in September 2002
Figure 5: Brazil Utility Sector: Assets Relative to Distress barrier

Figure 6. Estimated Actual Default Probability Versus Distance to Distress Utility Sector
Figure 7. Brazil Utility Sector: Implied Asset Volatility

(Std. deviation in percent)

Figure 8. Thailand: Assets Minus Distress Barrier: 1992 vs. 1996
Figure 9. Thailand: Distance to Distress by Sector: July 1997.

Figure 10. Thailand: Distance to Distress by Sector: October 1997.
Figure 11. Brazil: Convertibility Risk and Capital Outflows through CC5 Accounts

Convertibility risk
(Probability in percent, left scale)

Capital outflows
(in US$ million, right scale)

Mar-02  Jun-02  Sep-02  Dec-02

Figure 12. Thailand: Financial Sector Distance to Distress 1997

(Std. deviation from distress barrier)

July 1997  October 1997

Commercial Banks  Financial Cos.
Figure 13. Thailand: Value of the Financial Sector Guarantee 1997

(In percent of 1996 nominal GDP)
APPENDIX I. BLACK-SCHOLES OPTION PRICING FORMULA IN CONTINGENT CLAIMS
ANALYSIS

The contingent claims approach (CCA) defines fundamental relationships between the value of assets and the value of claims. CCA can be applied to simplified balance sheets with liabilities composed of senior debt and junior equity, or used with more complicated balance sheets that include senior collateralized debt and other classes of junior claims such as convertible securities and preferred equity. The purpose of CCA is to analyze how the value of these claims on firm assets change as the value of the firm changes through time. Here, we consider the simplified balance sheet of senior debt and junior equity and discuss the application of the Black-Scholes option pricing formula.

A. Assets and Liabilities as Implicit Options

The total market value of assets, $A$, of an firm financed with debt, $D$, and equity, $E$, is equal to the market value of equity plus market value of risky debt. Asset value is derived from the stochastic discounted present value of income minus expenditures with the potential for asset value to decline below the point where scheduled debt payments can be made. If assets fall to or below this level, then default is the result. This level is often referred to as distress barrier, $DB$ and represents the default-free value of debt.

The holders of equity are holders of a junior claim and have a contingent claim on the residual value of assets in the future. In this manner, the value of equity can be viewed as an option where holders of equity receive the maximum of either assets minus debt, or nothing in the case of default. The value of equity, therefore, is,

$$E = \max[A - DB, 0]. \quad (1)$$

The case of risky debt, however, is slightly more complex. Holders of debt are obligated to absorb losses if there is a default and the guarantee of repayment by the lender can be modeled as an implicit put option (i.e. in the event of default, the bondholders have a right to sell the remaining assets of the firm). Thus, holders of risky debt receive the minimum of the default-free value or, in the event of default, the senior claim on assets. Beginning with the relationship between risky and default-free debt, this can be modeled as,

$$\text{Value of default-free debt} = \text{Value of risky debt} + \text{Value of the guarantee}, \quad (2)$$

$$\text{Value of risky debt} = \text{value of default-free debt} - \text{implicit put option} \quad (3)$$

Since the value of default-free debt is the distress barrier and the implicit put option on the assets of the firm yields $\max[A - DB, 0]$, the market value of risky debt can be modeled as,
\[ D = \min [ A, DB ] = DB - \max [ DB - A, 0 ]. \quad (4) \]

Inserting these option pricing relationships into the economic balance sheet identity results in a value of firm assets at time \( t \) of,

\[ A = D + E, \quad (5) \]

\[ A = DB - \max [ DB - A, 0 ] + \max [ A - DB, 0 ]. \quad (6) \]

**B. Black-Scholes Formula**

Given that assets and liabilities in the firm balance sheet can be related using implicit options, the standard option pricing formula can be used to price these relationships. The main insight behind the methodology of the Black-Scholes formula is that the value of the option can be derived by forming a riskless hedge portfolio.\(^{43}\) A riskless portfolio is created consisting of a position in a derivative security and a position in a stock. The risk-free nature of the portfolio is derived from the fact that both the derivative and stock price are affected by the same underlying source of uncertainty. Over any short period of time, the two must be perfectly correlated. If the appropriate positions are established, the gain (loss) from the stock position always offsets the loss (gain) from the derivative security, so that the overall value of the position at the end of the period is known with certainty. Therefore, the return of the hedge portfolio is equal to the risk-free rate of interest. By forming a riskless hedge portfolio, the derivation relies primarily on no-arbitrage principles as opposed to equilibrium relationships. The return must be equal to the risk-free rate over the holding period.

Using the Black-Scholes formula, the value of equity as a call option on firm assets is,

\[ E = AN(d_1) - DBe^{-rT} N(d_2), \quad (7) \]

where \( A \) is the value of the assets, \( E \) is the value of equity, \( DB \) is the distress barrier or value of default-free debt, \( r \) is the risk-free rate of interest, \( T \) is the time to maturity on the default-free bond in years. \( N(d) \) is the cumulative probability distribution function for a standard normal variable (i.e. the probability that a random draw from a standard normal distribution will be below \( d \)) where,

\(^{43}\) For readers interested in a more explicit derivation of the Black-Scholes option pricing formula, see Black and Scholes (1973) and Merton (1973, 1974). While the derivations in these studies use continuous-time mathematics, Hull (1993) and Baxter and Rennie (1996) detail how binomial models can be used to develop discrete-time representations.
\[
d_1 = \frac{\ln\left(\frac{A}{DB}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}}
\]
\[
d_2 = d_1 - \sigma_A \sqrt{T},
\]

and \(\sigma_A\) is the standard deviation of return on firm assets. Following a similar process, the value of the implicit put option from the guarantee on debt is,

\[
P = DBe^{-rTN(-d_2)} - AN(-d_1),
\]

where \(P\) is the put option and remaining variables are defined as above. The implied market value of risky firm debt is the value of the implicit put option plus the distress barrier. The value of the put option can also be viewed as the expected loss if default occurs. In this way, the credit spread is the risk premium required by the bondholders to compensate for the expected loss. That is,

\[
Spread = -\frac{1}{T} \ln \left[ N(d_2) + \frac{A}{DB} e^{-rTN(-d_1)} \right].
\]

Each of the Black-Scholes formulas above contain two unknowns, firm assets and volatility of firm assets. The relationship between volatility of firm assets and volatility of equity is given by,

\[
E = \frac{\sigma_A}{\sigma_E} AN(d_1),
\]

where \(\sigma_E\) is the standard deviation of equity.\(^{44}\) Here, \(N(d_1)\) is the change in the price of equity with respect to a change in the underlying assets of the firm, or \(\partial E/\partial A\). This ratio is also referred to as the option \textit{delta}. However, the main implication of the above relationship is that the standard deviation of equity can be derived from historical data and used to solve for asset volatility. If historical data is not viewed as a good predictor of future asset volatility and if local derivative markets were well developed, then an alternative method would be to use individual firm equity option data to derive the implied volatility of equity. In addition, option data on an overall equity market index and firm or sectoral betas could be combined to derive implied equity volatility at the firm or sectoral levels.

\(^{44}\) See Hull (1993, p. 38).
C. Example of Distance to Distress and Probability of Default

This section provides a brief numerical example of estimating distance to distress and probability of default. Assume that using the option pricing relationships developed above and the volatility of firm assets, a firm can be characterized by the following,

- Asset Value = 1000,
- Annualized Asset Volatility = $\sigma_A = 0.36$,
- Distress barrier = 600,
- Risk-free Rate = 0.04,

Distance to distress expressed in relation to the standard deviation of firm assets over the time horizon $T$ is measure through evaluation of $d_1$,

$$\text{Distance to distress} = \frac{\ln\left(\frac{1000}{600}\right) + \left(0.05 + \frac{0.13}{2}\right)}{0.36} = 1.7. \quad (12)$$

Thus, the distance to distress over a one-year horizon for a firm with these characteristics is 1.7 standard deviations of firm asset value from the distress barrier. Converting this measure to a probability of default requires calculating the cumulative normal distribution function, $N(.)$. This can be done using numerical methods or polynomial approximation. Tables that compute $N(.)$ are also found in many financial and econometric texts. Using one of these methods will yield the probability of default as,

$$\text{Probability of Default} = N(-1.7) = 0.04 \text{ or } 4\% \quad (13)$$

As discussed in the text, however, the probability of default as calculated above relies on risk-neutral pricing and generally overstates the true probability of default. Moody’s KMV uses actual historical data on corporate defaults to map the distance to distress in (12) to an Estimated Default Frequency™ (EDF). Nevertheless, the example provided here is instructive of the procedure as a whole.

D. Properties of Black-Scholes

When firm assets become very large, both $d_1$ and $d_2$ also become very large, meaning $N(d_1)$ and $N(d_2)$ are close to 1. When this happens, equity behaves more like a forward contract with delivery price equal to the distress barrier. The price of equity becomes,

$$E = A - D_e e^{-rT}. \quad (14)$$

As firm assets become very large, the price of the implicit put option from the guarantee on debt approaches zero since $N(d_1)$ and $N(d_2)$ both approach zero. This is merely a recognition
of the fact that risky debt approaches default-free debt as assets of the firm continue to grow. The likelihood that default will occur, necessitating bondholders to exercise the put option on firm assets, becomes very low. Conversely, as firm assets decline, both \( d_1 \) and \( d_2 \) also decline, meaning \( N(d_1) \) and \( N(d_2) \) approach zero and equity becomes increasingly worthless. At the same time, however, the implicit put option from the guarantee on debt becomes more valuable since \( N(d_1) \) and \( N(d_2) \) both approach 1.

As volatility of firm assets approaches zero, both \( d_1 \) and \( d_2 \) become very large and \( N(d_1) \) and \( N(d_2) \) approach 1. As volatility declines to zero, the firm looks more like a riskless asset and the value of equity looks more like a forward contract, as was the case under very large firm assets. As firm asset volatility rises, the implicit put option on debt becomes more valuable since the probability that firm assets fall below the distress barrier increases. Since firm assets are fewer standard deviations away from the distress barrier, the distance to distress decreases, making equity less valuable and the implicit put option more valuable.
APPENDIX II. MULTI-SECTOR CCA MODEL

The same general principals of CCA as applied to firm or industry level balance sheets can also be applied to the public and financial sectors to construct interrelated consolidated balance sheets. The multi-sector CCA balance sheets are displayed in Figure A1. The liabilities of a sector are contingent claims on the assets of the sector. Corporate sector liabilities are risky debt and equity which are contingent claims on corporate sector assets. Financial sector liabilities – debt, deposits and equity – are contingent claims on banking sector assets including loans, reserves and other assets. The major public sector liabilities are guarantees to “too-big-to-fail” entities, foreign debt, local currency debt and money. These represent contingent claims on public sector assets.

Risk can be transmitted from one sector to another via value changes risky debt, guarantees, and junior claims which are linked across sectors. For example, debt obligations exist between corporate sectors and banks, between the sovereign and banks. Assets of banks include risky debt to the corporate sector and sovereign debt. Changes in assets of one sector lead to changes in value of liabilities of that sector which in turn affect the value of assets, and liabilities, in other sectors. Foreign and domestic investors hold debt and equity claims in these various sectors. Changes in prices – exchange rate, interest rate, and other prices – affect the value of assets, and liabilities, and thus have an impact on risk transmission between sectors. While only three sectors are shown in Figure A1 for simplicity of exposition, a typical Moody’s-MfRisk macrofinancial model has about 15 corporate industry sectors, 5 to 10 bank and financial sectors and one sovereign sector made up of a combined market value balance sheet of the government and monetary authorities.

Within the financial sector, CCA can be applied to sub-sectors including banks, pension funds, and insurance companies. Like the non-financial corporate sector, equity of the financial sector is a contingent claim on total assets and is modeled as a call option with strike price equal to the default-free value of debt. This relationship is used to derive the implied market value of financial sector assets which is then used to derive the value of the financial guarantee from the public sector. The value of this financial guarantee can be modeled as a put option, giving the financial sector the “right to sell” firm assets to the government sector if financial sector assets fall below the distress barrier. As the health of the financial sector declines, the market value of assets approaches the distress barrier resulting in a lower distance to distress and higher value on the financial guarantee.

Application of CCA to the public sector is more complex since the public sector combines the functions of the government and monetary authorities. The goal is to construct the liability side of the balance sheet so that the liabilities can be valued and linked to the value of total assets. Securities issued by the public sector and the financial guarantees provided by it give the holders contingent claims on the assets of the government and monetary authorities.

45 The value of the put option should be adjusted to reflect partial recapitalization instead of full recapitalization by assuming an estimated recover rate below unity.
To apply CCA to combined balance sheet of the government and monetary authorities, it is useful to begin by defining eligible assets and liabilities. The main entries on the asset side of the public sector balance sheet include: (i) foreign currency reserves; (ii) the present value of future taxes and revenues; (iii) public assets such as equity in public enterprises, land, and mineral assets; and (iv) value of the public sector’s monopoly on the issue of money. The main entries on the liability side of the balance sheet include: (i) the present value of government expenditures, including social insurance and entitlement programs; (ii) domestic currency debt; (iii) foreign currency or external debt; (iv) relevant financial guarantees; and (v) base money.

Establishing the interrelationship and priority within assets and liabilities is essential in applying CCA to the public sector balance sheet. While the priority of debt service obligations varies among countries and over time within any one country, this analysis assumes the most senior liabilities of the public sector are foreign currency debt plus financial guarantees. Foreign currency debt is viewed as senior since experience indicates that governments take exceptional steps to meet such payments, subjugating other elements of the policy agenda. Domestic currency debt and the guarantees of currency convertibility are then junior claims in the liability structure. Based on this ordering, sustained declines in the value of public sector assets would lead initially to restrictions on currency convertibility and a restructure of domestic currency debt in order to maintain payments on foreign-currency debt. However, if public sector assets continue to decline value, a default on foreign currency debt becomes more likely.

In the MfRisk model, public sector liabilities are contingent claims modeled with implicit options. The outstanding amounts of domestic currency debt of the government and base money are modeled as a call option on total public sector assets with the distress barrier composed of senior foreign currency debt and financial guarantees. As discussed above, the

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47 The total foreign reserves of the public sector in the MfRisk model include actual reserves plus contingent reserves from international financial institutions like the IMF or contingent credit lines.
48 Liabilities only refer to those held outside of the combined government and monetary authorities.
49 Changing the ordering of priority does not eliminate the use of CCA, but instead simply re-ordersthe option pricing relationships.
50 Base money is issued by the monetary authority with the associated obligation to exchange base money for foreign currency reserves in the absence of capital controls. When domestic currency is exchanged for foreign currency, foreign currency reserves of the banking system and monetary authority are reduced.
51 In some cases, market participants have traded protection against the imposition of capital controls which restrict convertibility of domestic currency. For example, Tavakoli (2001) examined convertibility protection prices and found that they often trade at 50 to 100 percent higher than credit default protection, indicating that participants feel convertibility risk is greater than default risk.
52 The balance sheet of the public sector is the only balance sheet that contains accounts denominated in different currencies. In order to compare claims in a common currency, the MfRisk model takes the value of (base money + debt in local currency + interest costs)*(FXrate) to represent the “equity” call option on public sector assets. The distress barrier is composed of foreign currency denominated
financial guarantee is modeled as a put option using assets and the distress barrier of the financial sector. The implied market value of external debt is modeled as the default-free value minus an expected loss which is modeled as an implicit put option (debtholders receive the minimum of the default-free value or, in the event of default, lower value after debt restructuring).

The balance sheet of the public sector can be constructed using pricing information from the international financial market. Using this data, the Mf/Risk model can be used to derive the implied asset value of the public sector. For example, if measured in U.S. dollars, the Mf/Risk model uses the value and volatility of the junior claims of the sovereign (i.e. money and local currency debt) in dollar terms. When based in dollars, the volatility of the exchange rate becomes an important element. The book value of short-term and long-term sovereign foreign debt in dollar terms gives the public sector foreign debt distress barrier. Using the sovereign contingent claim balance sheet relationships, this allows for the calculation of the implied sovereign asset value and the implied volatility of sovereign assets. For example, for Brazil in early 2003 the value of the junior claims of the sovereign in dollar terms was US$104 billion and its volatility in dollar terms was 0.98. The book value of foreign and dollar linked debt give a default barrier of $100 billion. Using this information, and two finance equations, the implied sovereign asset value was $204 billion and implied sovereign asset volatility is estimated to be 0.6.

Therefore, changes in the distress barrier for the public sector come from two sources: (i) changes in the liability structure of external debt from, for example, changes in maturity structure or currency movements; or, (ii) changes in the value of the financial guarantee due to changes in the health of the financial system. The implied market value of external debt is modeled as the default-free value minus a put option since debtholders receive the minimum of the default-free value or, in the event of default, the senior claim on public sector assets.

For example, the financial market volatility in Brazil during 2002 had implications for sustainability of the public sector. As the currency depreciated, the cost of both external debt and domestic dollar-linked debt increased. Furthermore, in response to increased inflation as a result of the exchange rate pass through, the central bank raised the policy interest rate by over 700 basis points. This also translated into higher debt costs through the stock of interest-rate indexed debt. Overall, the declined in international reserves reduced public sector assets and the debt dynamics increased the distress barrier, lowering the public sector distance to distress (Figures A2 and A3).54

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53 In the case of Brazil, both domestic dollar-linked sovereign debt and external dollar-denominated debt are used to construct the distress barrier.  
54 Corporate sector distance to distress is mapped into estimated actual probability of default based on the extensive historical database of corporate defaults. Since no similar record exists at the sovereign level, Mf/Risk uses various financial market relationships including credit default swaps to map distance to distress into estimated actual sovereign default probabilities.
Figure A1. Multi-Sector CCA Balance Sheets

### Non-financial Corporate Sector

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Sector Assets</td>
<td>Debt</td>
</tr>
<tr>
<td></td>
<td>Equity</td>
</tr>
</tbody>
</table>

### Financial Sector

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans and other Assets (including loans to corporate sector and public sector)</td>
<td>Debt / Deposits / Liabilities (default-free value minus put option)</td>
</tr>
<tr>
<td>Financial Guarantees (modeled as put option)</td>
<td>Equity (call option on financial sector total assets)</td>
</tr>
</tbody>
</table>

### Combined Government and Monetary Authorities

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign Currency</td>
<td>Foreign-currency Debt (default-free value - put option)</td>
</tr>
<tr>
<td>Net Fiscal Asset</td>
<td>Financial Guarantees (put options related to financial sector)</td>
</tr>
<tr>
<td>Value of Monopoly on Issue of Money</td>
<td>Base Money and Local-currency Debt (call options on public sector assets)</td>
</tr>
<tr>
<td>Other Public Assets</td>
<td></td>
</tr>
</tbody>
</table>
Figure A2. Brazil: Public Sector Distance to Distress

Figure A3. Brazil: Public Sector Probability of Default

Source: Moody's M/Risk Model.